Hello SHMS students! This summer math packet is intended for preparing for your upcoming 7th grade Algebra math class. Work through this packet in order; in every section there are notes for each main concept with examples and explanations. At the end of every section there is a mixed practice. If you get stuck or need MORE review, go to khanacademy.org and search through the 7th Grade Pre-Algebra level for the specific topic. There you will find helpful videos, more practice problems and quizzes. A digital answer key can be found on the SHMS website!

These concepts will be covered early in the fall, so let's get a head start and be prepared to learn new and exciting things! Have a great summer!

The 6th grade mathematics teachers

SECTION I: Real Number System & Integer Operations

You should review the properties of the classification of real numbers.

Example:

Look at the numbers $^-3.8, \frac{13}{19}$, 0, $^-\pi$, $\sqrt{3}$, 57, $^-14$.

Natural:	57	numbers used to count
Whole:	0, 57	natural numbers and zero
Integers:	0, 57, $^-14$	whole numbers and their opposites
Rational:	0, 57, $^-14$, $^-3.8$, $\frac{13}{19}$	integers and terminating and nonrepeating decimals
Irrational:	$^-\pi$, $\sqrt{3}$	infinite, nonrepeating decimals
Real:	0, 57, $^-14$, $^-3.8$, $^-\pi$, $\sqrt{3}$, $\frac{13}{19}$	rational and irrational numbers

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Order of Operations

Use the **order of operations** to evaluate numerical expressions.

1. Evaluate the expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example 1 **Evaluate $(10 - 2) - 4 \cdot 2$.**

$(10 - 2) - 4 \cdot 2 = 8 - 4 \cdot 2$	Subtract first since 10 − 2 is in parentheses.
$= 8 - 8$	Multiply 4 and 2.
$= 0$	Subtract 8 from 8.

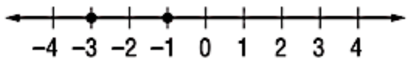
Example 2 **Evaluate $8 + (1 + 5)^2 \div 4$.**

$8 + (1 + 5)^2 \div 4 = 8 + 6^2 \div 4$	First, add 1 and 5 inside the parentheses.
$= 8 + 36 \div 4$	Find the value of 6^2.
$= 8 + 9$	Divide 36 by 4.
$= 17$	Add 8 and 9.

Comparing and Ordering Integers

When two numbers are graphed on a number line, the number to the left is always less than (<) the number to the right. The number to the right is always greater than (>) the number to the left.

Model



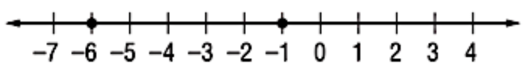
Words -3 is less than -1. -1 is greater than -1.

Symbols $-3 < -1$ $-1 > -3$

The symbol points to the lesser number.

Example 1 **Replace the ● with < or > to make −1 ● −6 a true sentence.**

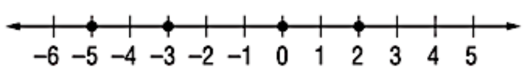
Graph each integer on a number line.



Since -1 is to the right of -6, $-1 > -6$.

Example 2 **Order the integers 2, −3, 0, −5 from least to greatest.**

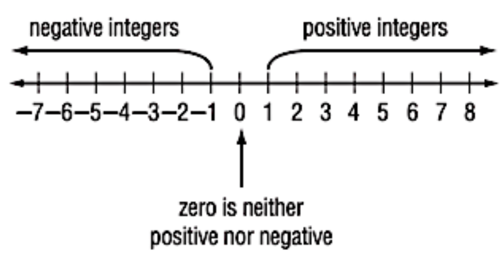
To order the integers, graph them on a number line.



Order the integers by reading from left to right: $-5, -3, 0, 2$.

Integers and Absolute Value

Integers less than zero are **negative integers**. Integers greater than zero are **positive integers**.



The absolute value of an integer is the distance the number is from zero on a number line. Two vertical bars are used to represent absolute value. The symbol for absolute value of 3 is $|3|$.

Example 1 **Write an integer that represents 160 feet below sea level.**

Because it represents *below* sea level, the integer is -160.

Example 2 **Evaluate $|-2|$.**

On the number line, the graph of -2 is 2 units away from 0. So, $|-2| = 2$.

Adding Integers

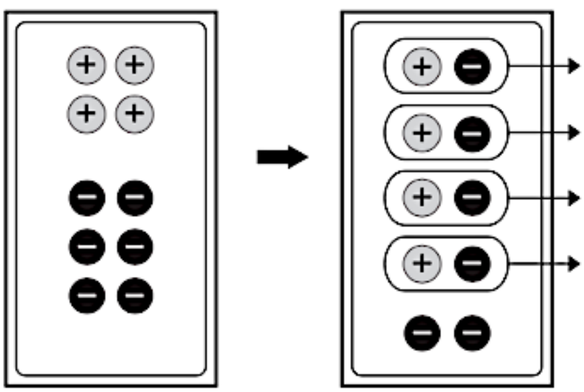
For integers with the same sign:
- the sum of two positive integers is positive.
- the sum of two negative integers is negative.

For integers with different signs, subtract their absolute values. The sum is:
- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

To add integers, it is helpful to use counters or a number line.

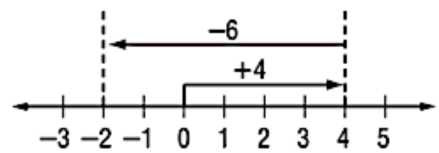
Example **Find $4 + (-6)$.**

Method 1 Use counters.
Combine a set of 4 positive counters and a set of 6 negative counters on a mat.



$4 + (-6)$ $\qquad$ $4 + (-6) = -2$

Method 2 Use a number line.
- Start at 0.
- Move 4 units right.
- Then move 6 units left.



$4 + (-6) = -2$

Subtracting Integers

To subtract an integer, add its opposite.

Example 1 **Find $6 - 9$.**

$$\begin{aligned} 6 - 9 &= 6 + (-9) && \text{To subtract 9, add } -9. \\ &= -3 && \text{Simplify.} \end{aligned}$$

Example 2 **Find $-10 - (-12)$.**

$$\begin{aligned} -10 - (-12) &= -10 + 12 && \text{To subtract } -12, \text{ add 12.} \\ &= 2 && \text{Simplify.} \end{aligned}$$

Example 3 **Evaluate $a - b$ if $a = -3$ and $b = 7$.**

$$\begin{aligned} a - b &= -3 - 7 && \text{Replace } a \text{ with } -3 \text{ and } b \text{ with 7.} \\ &= -3 + (-7) && \text{To subtract 7, add } -7. \\ &= -10 && \text{Simplify.} \end{aligned}$$

SECTION I PRACTICE: **Real Number System & Integer Operations**

Evaluate each expression using the order of operations, show your work:

1. $5 \cdot 2^2 + 32 \div 8$

2. $(5 + 7)^2 \div 12$

3. $|12| \div 2 \cdot |-5|$

4. $9 + |6| \div 1^2$

Write an integer for each situation:

5. a profit of $12

6. 1,440 feet below sea level

7. $22°\text{F}$ below 0

8. a gain of 31 yards

Order each from LEAST to GREATEST:

9. $-1, 5, -3 \; and \; 2$

10. $0, -4, -2 \; and \; 7$

11. $-3, |-2|, 4, 0 \; and \; 5$

Add or Subtract the integers:

12. $18 + (-12) + 5$

13. $-22 + (-10) + 15$

14. $7 - 9$

15. $-75 - 50$

16. $25 - (-14)$

17. $-8 - (-6)$

SECTION II: Rational Numbers, Fraction Operations & Proportions

Comparing and Ordering Rational Numbers

To compare fractions, rewrite them so they have the same denominator. The **least common denominator (LCD)** of two fractions is the LCM of their denominators.

Another way to compare fractions is to express them as decimals. Then compare the decimals.

Example 1 **Which fraction is greater, $\frac{3}{4}$ or $\frac{4}{5}$?**

Method 1 Rename using the LCD.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$$

The LCD is 20.

Because the denominators are the same, compare numerators.

Since $\frac{16}{20} > \frac{15}{20}$, then $\frac{4}{5} > \frac{3}{4}$.

Method 2 Write each fraction as a decimal. Then compare decimals.

$$\frac{3}{4} = 0.75$$

$$\frac{4}{5} = 0.8$$

Since $0.8 > 0.75$, then $\frac{4}{5} > \frac{3}{4}$.

Adding and Subtracting Fractions

Like fractions are fractions that have the same denominator. To add or subtract like fractions, add or subtract the numerators and write the result over the denominator.

Simplify if necessary.

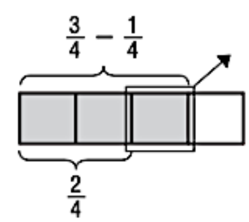
To add or subtract *unlike fractions*, rename the fractions with a least common denominator. Then add or subtract as with like fractions.

Example 1 **Subtract $\frac{3}{4} - \frac{1}{4}$. Write in simplest form.**

$\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4}$ Subtract the numerators.

$= \frac{2}{4}$ Write the difference over the denominator.

$= \frac{1}{2}$ Simplify.



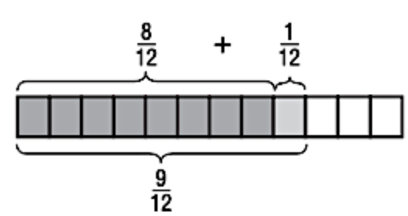
Example 2 **Add $\frac{2}{3} + \frac{1}{12}$. Write in simplest form.**

The least common denominator of 3 and 12 is 12.

$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ Rename $\frac{2}{3}$ using the LCD.

$$\begin{array}{rcr} \frac{2}{3} & \rightarrow & \frac{8}{12} \\ +\frac{1}{12} & \rightarrow & +\frac{1}{12} \\ \hline & & \frac{9}{12} \text{ or } \frac{3}{4} \end{array}$$

Add the numerators and simplify.



Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers:

1. Add or subtract the fractions. Rename using the LCD if necessary.
2. Add or subtract the whole numbers.
3. Simplify if necessary.

Example 1 **Find $14\frac{1}{2} + 18\frac{2}{3}$.**

$$\begin{array}{rcr} 14\frac{1}{2} & \rightarrow & 14\frac{3}{6} \\ +18\frac{2}{3} & \rightarrow & +18\frac{4}{6} \\ \hline & & 32\frac{7}{6} \text{ or } 33\frac{1}{6} \end{array}$$

Rename the fractions.

Add the whole numbers and add the fractions.

Simplify.

Example 2 **Find $21 - 12\frac{5}{8}$.**

$$\begin{array}{rcr} 21 & \rightarrow & 20\frac{8}{8} \\ -12\frac{5}{8} & \rightarrow & -12\frac{5}{8} \\ \hline & & 8\frac{3}{8} \end{array}$$

Rename 21 as $20\frac{8}{8}$.

First subtract the whole numbers and then the fractions.

Multiplying Fractions and Mixed Numbers

To multiply fractions, multiply the numerators and multiply the denominators.

$$\frac{5}{6} \times \frac{3}{5} = \frac{5 \times 3}{6 \times 5} = \frac{15}{30} = \frac{1}{2}$$

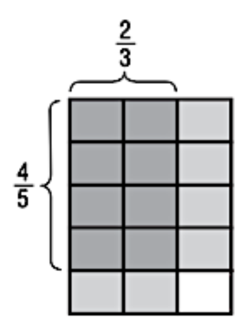
To multiply mixed numbers, rename each mixed number as a fraction. Then multiply the fractions.

$$2\frac{2}{3} \times 1\frac{1}{4} = \frac{8}{3} \times \frac{5}{4} = \frac{40}{12} = 3\frac{1}{3}$$

Example 1 **Find $\frac{2}{3} \times \frac{4}{5}$. Write in simplest form.**

$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$ ← Multiply the numerators. ← Multiply the denominators.

$= \frac{8}{15}$ Simplify.



Example 2 **Find $\frac{1}{3} \times 2\frac{1}{2}$. Write in simplest form.**

$\frac{1}{3} \times 2\frac{1}{2} = \frac{1}{3} \times \frac{5}{2}$ Rename $2\frac{1}{2}$ as an improper fraction, $\frac{5}{2}$.

$= \frac{1 \times 5}{3 \times 2}$ Multiply.

$= \frac{5}{6}$ Simplify.

Dividing Fractions and Mixed Numbers

To divide by a fraction, multiply by its multiplicative inverse or reciprocal. To divide by a mixed number, rename the mixed number as an improper fraction.

Example 1 **Find $3\frac{1}{3} \div \frac{2}{9}$. Write in simplest form.**

$3\frac{1}{3} \div \frac{2}{9} = \frac{10}{3} \div \frac{2}{9}$	Rename $3\frac{1}{3}$ as an improper fraction.
$= \frac{10}{3} \cdot \frac{9}{2}$	Multiply by the reciprocal of $\frac{2}{9}$, which is $\frac{9}{2}$.
$= \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}}$	Divide out common factors.
$= 15$	Multiply.

Algebra: Solving Proportions

A **proportion** is an equation stating that two ratios are equivalent. Since rates are types of ratios, they can also form proportions. In a proportion, a **cross product** is the product of the numerator of one ratio and the denominator of the other ratio.

Example 1 **Determine whether $\frac{2}{3}$ and $\frac{10}{15}$ form a proportion.**

$\frac{2}{3} \stackrel{?}{=} \frac{10}{15}$	Write a proportion.
$2 \times 15 \stackrel{?}{=} 3 \times 10$	Find the cross products.
$30 = 30$ ✓	Multiply.

The cross products are equal, so the ratios form a proportion.

Example 2 **Solve $\frac{8}{a} = \frac{10}{15}$.**

$\frac{8}{a} = \frac{10}{15}$	Write the proportion.
$8 \times 15 = a \times 10$	Find the cross products.
$120 = 10a$	Multiply.
$\frac{120}{10} = \frac{10a}{10}$	Divide each side by 10.
$12 = a$	Simplify.

The solution is 12.

<u>SECTION II PRACTICE</u>: **Rational Numbers, Fraction Operations & Proportions**

Determine if the number is rational; EXPLAIN your reasoning:

1. $\frac{8}{21}$

2. $0.5055055\ldots$

Order each from LEAST to GREATEST:

3. $63\%, \frac{2}{3}\ and\ 0.65$

4. $0.2, 2\%$ and $\frac{1}{12}$

Add, Subtract, Multiply or Divide the fractions, show all your work:

5. $\frac{5}{9}+\frac{5}{6}$

6. $\frac{2}{5}-\frac{1}{3}$

7. $\frac{1}{2}-\frac{7}{9}$

8. $4+\frac{8}{9}$

9. $18\frac{1}{2}+5\frac{5}{5}$

10. $9-3\frac{2}{5}$

11. $\frac{1}{2}\cdot\frac{7}{8}$

12. $\frac{1}{3}\div\frac{3}{5}$

13. $6\frac{2}{3}\div 3\frac{1}{9}$

14. $2\frac{1}{3}\cdot\frac{4}{6}$

Solve each proportion, show all your work:

15. $\frac{2.8}{7.7}=\frac{z}{4.4}$

16. $\frac{5}{15}=\frac{15}{x}$

SECTION III: **Expressions & Equations**

Algebra: Variables and Expressions

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

Example 1 **Evaluate $6x - 7$ if $x = 8$.**

$6x - 7 = 6(8) - 7$	Replace *x* with 8.
$= 48 - 7$	Use the order of operations.
$= 41$	Subtract 7 from 48.

Example 2 **Evaluate $5m - 3n$ if $m = 6$ and $n = 5$.**

$5m - 3n = 5(6) - 3(5)$	Replace *m* with 6 and *n* with 5.
$= 30 - 15$	Use the order of operations.
$= 15$	Subtract 15 from 30.

Example 3 **Evaluate $\frac{ab}{3}$ if $a = 7$ and $b = 6$.**

$\frac{ab}{3} = \frac{(7)(6)}{3}$	Replace *a* with 7 and *b* with 6.
$= \frac{42}{3}$	The fraction bar is like a grouping symbol.
$= 14$	Divide.

Solving Addition and Subtraction Equations

Remember, equations must always remain balanced. If you subtract the same number from each side of an equation, the two sides remain equal. Also, if you add the same number to each side of an equation, the two sides remain equal.

Example 1 **Solve $x + 5 = 11$. Check your solution.**

$x + 5 = 11$	Write the equation.
$\underline{\quad - 5 = -5}$	Subtract 5 from each side.
$x \quad = 6$	Simplify.

Check $x + 5 = 11$	Write the equation.
$6 + 5 \stackrel{?}{=} 11$	Replace *x* with 6.
$11 = 11$ ✓	This sentence is true.

The solution is 6.

Example 2 **Solve $15 = t - 12$. Check your solution.**

$15 = t - 12$	Write the equation.
$\underline{+12 = \quad +12}$	Add 12 to each side.
$27 = t$	Simplify.

Check $15 = t - 12$	Write the equation.
$15 \stackrel{?}{=} 27 - 12$	Replace *t* with 27.
$15 = 15$ ✓	This sentence is true.

The solution is 27.

Solving Multiplication Equations

If each side of an equation is divided by the same non-zero number, the resulting equation is equivalent to the given one. You can use this property to solve equations involving multiplication and division.

Example 1 **Solve $45 = 5x$. Check your solution.**

$45 = 5x$	Write the equation.
$\frac{45}{5} = \frac{5x}{5}$	Divide each side of the equation by 5.
$9 = x$	$45 \div 5 = 9$

Check	$45 = 5x$	Write the original equation.
	$45 \stackrel{?}{=} 5(9)$	Replace *x* with 9. Is this sentence true?
	$45 = 45$ ✓	

The solution is 9.

Example 2 **Solve $-21 = -3y$. Check your solution.**

$-21 = -3y$	Write the equation.
$\frac{-21}{-3} = \frac{-3y}{-3}$	Divide each side by -3.
$7 = y$	$-21 \div (-3) = 7$

Check	$-21 = -3y$	Write the original equation.
	$-21 \stackrel{?}{=} -3(7)$	Replace *y* with 7. Is this sentence true?
	$-21 = -21$ ✓	

The solution is 7.

Solving Two-Step Equations

To solve two-step equations, you need to add or subtract first. Then divide to solve the equation.

Example 1 **Solve $7v - 3 = 25$. Check your solution.**

$7v - 3 = 25$	Write the equation.
$+3 = +3$	Add 3 to each side.
$7v = 28$	Simplify.
$\frac{7v}{7} = \frac{28}{7}$	Divide each side by 7.
$v = 4$	Simplify.

Check	$7v - 3 = 25$	Write the original equation.
	$7(4) - 3 \stackrel{?}{=} 25$	Replace *v* with 4.
	$28 - 3 \stackrel{?}{=} 25$	Multiply.
	$25 = 25$ ✓	The solution checks.

The solution is 4.

Example 2 **Solve $-10 = 8 + 3x$. Check your solution.**

$-10 = 8 + 3x$	Write the equation.
$-8 = -8$	Subtract 8 from each side.
$-18 = 3x$	Simplify.
$\frac{-18}{3} = \frac{3x}{3}$	Divide each side by 3.
$-6 = x$	Simplify.

Check	$-10 = 8 + 3x$	Write the original equation.
	$-10 \stackrel{?}{=} 8 + 3(-6)$	Replace *x* with -6.
	$-10 \stackrel{?}{=} 8 + (-18)$	Multiply.
	$-10 = -10$ ✓	The solution checks.

The solution is -6.

Rate of Change and Slope

- A rate of change is a rate that describes how one quantity changes in relation to another.
- Slope tells how steep the line is.
- Slope is given by the formula $\frac{\text{change in } y}{\text{change in } x}$ or $\frac{\text{vertical change}}{\text{horizontal change}}$.

Example 1 **Find the rate of change for the table.**

Students	Number of Textbooks
5	15
10	30
15	45
20	60

The change in the number of textbooks is 15 while the change in the number of students is 5.

$$\frac{\text{change in number of textbooks}}{\text{change in number of students}} = \frac{15 \text{ textbooks}}{5 \text{ students}}$$ The number of textbooks increased by 15 for every 5 students.

$$= \frac{3 \text{ textbooks}}{1 \text{ student}}$$ Write as a unit rate.

So, the number of textbooks increases by 3 textbooks per student.

SECTION III PRACTICE: Expressions & Equations

*Evaluate each expression if **r = 5, s = 2, t = 7** and **u = -1**, show all your work:*

1. $11\text{u} - 7$

2. $2\text{t}^2 + 18$

3. $\frac{(3+u)^2}{8}$

4. $4r - 10s$

*Evaluate each expression if **a = 4.1, b = 5.7,** and **c = 0.3**, show all your work:*

5. $\text{a} + \text{c} - \text{b}$

6. $10 - (\text{a} + \text{b})$

Solve the following equations, show all your work:

7. $12 + y = 15$

8. $-14 + t = 26$

9. $c - 3.5 = -6.4$

10. $-144 = -9r$

11. $-9x - 10 = 62$

12. $1 - n = 11$

13. $-5b + 12 = 2$

14. $19 + 13x = 42$

Find the rate of change (slope) for each table:

15.

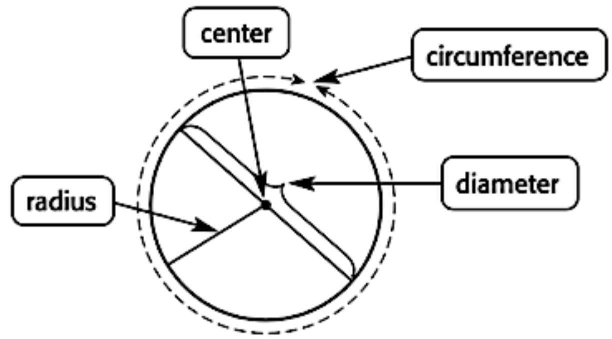
Side Length	Perimeter
1	4
2	8
3	12
4	16

16.

Days	Plant Height (in.)
7	4
14	11
21	18
28	25

SECTION IV: Circles, Circumference, Areas and Volumes

A **circle** is the set of all points in a plane that are the same distance from a given point, called the **center**. The **diameter** d is the distance across the circle through its center. The **radius** r is the distance from the center to any point on the circle. The **circumference** C is the distance around the circle. The circumference C of a circle is equal to its diameter d times π, or 2 times its radius r times π.



Example 1 **Find the circumference of a circle with a diameter of 7.5 centimeters.**

$C = \pi d$	Circumference of a circle.
$C \approx 3.14 \times 7.5$	Replace π with 3.14 and d with 7.5.
$C \approx 23.55$	The circumference of the circle is about 23.55 centimeters.

Example 2 **If the radius of a circle is 14 inches, what is its circumference?**

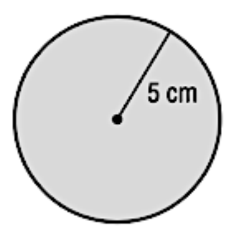
$C = 2\pi r$	
$C \approx 2 \times 3.14 \times 14$	Replace π with 3.14 and r with 14.
$C \approx 87.92$	The circumference of the circle is about 87.92 inches.

Area of Circles

The area A of a circle equals the product of pi (π) and the square of its radius r.

$$A = \pi r^2$$

Example 1 **Find the area of the circle.**



$A = \pi r^2$	Area of circle
$A \approx 3.14 \cdot 5^2$	Replace π with 3.14 and r with 5.
$A \approx 78.5$	

The area of the circle is approximately 78.5 square centimeters.

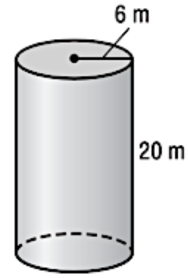
Example 2 **Find the area of a circle that has a diameter of 9.4 millimeters.**

$A = \pi r^2$	Area of a circle
$A \approx 3.14 \cdot 4.7^2$	Replace π with 3.14 and r with $9.4 \div 2$ or 4.7.
$A \approx 69.4$	

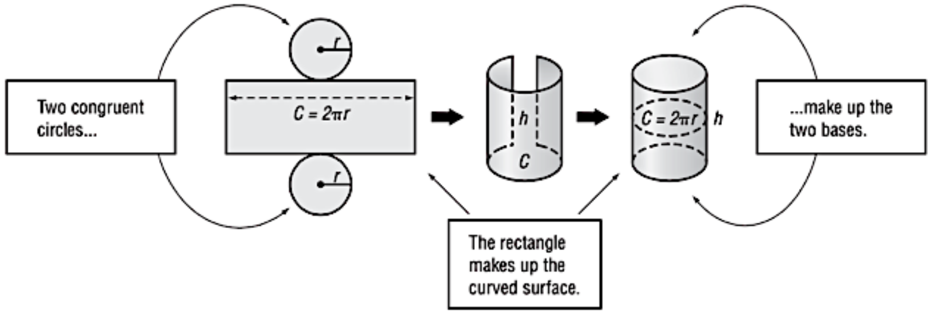
The area of the circle is approximately 69.4 square millimeters.

Surface Area of Cylinders

The diagram below shows how you can put two circles and a rectangle together to make a cylinder.

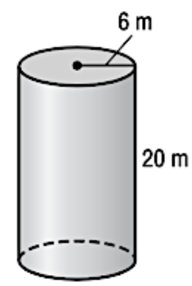


$$\underbrace{S}_{\text{The surface area of a cylinder}} \underbrace{=}_{\text{equals}} \underbrace{2(\pi r^2)}_{\text{the area of two bases}} \underbrace{+}_{\text{plus}} \underbrace{(2\pi r)h}_{\text{the area of the curved surface.}}$$



In the diagram above, the length of the rectangle is the same as the circumference of the circle. Also, the width of the rectangle is the same as the height of the cylinder.

Example **Find the surface area of the cylinder. Use 3.14 for π. Round to the nearest tenth.**



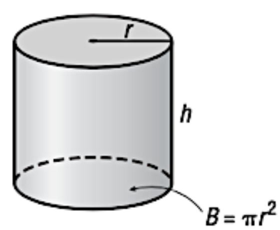
$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder.
$S = 2\pi(6)^2 + 2\pi(6)(20)$	Replace π with 3.14, r with 6, and h with 20.
≈ 979.7	Simplify.

The surface area is about 979.7 square meters.

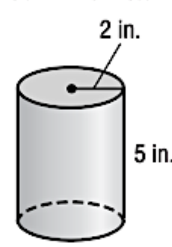
Volume of Cylinders

As with prisms, the area of the base of a **cylinder** tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder. The volume V of a cylinder with radius r is the area of the base B times the height h.

$V = Bh$ or $V = \pi r^2 h$, where $B = \pi r^2$



Example **Find the volume of the cylinder. Use 3.14 for π. Round to the nearest tenth.**



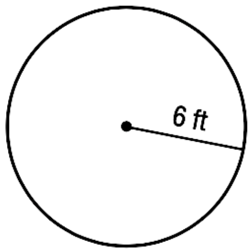
$V = \pi r^2 h$	Volume of a cylinder
$V \approx 3.14(2)^2(5)$	Replace π with 3.14, r with 2, and h with 5.
$V \approx 62.8$	Simplify.

The volume is approximately 62.8 cubic inches. Check by using estimation.

<u>SECTION IV:PRACTICE</u>: Circles: Circumference, Areas and Volumes

*Find the **circumference** of each circle; use 3.14 for π.*

1.



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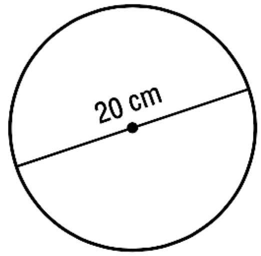

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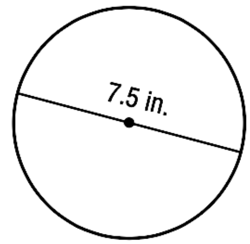
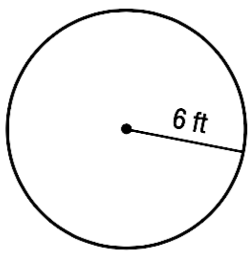


*Find the **AREA** of each circle; use 3.14 for π, show all your work:*

4.



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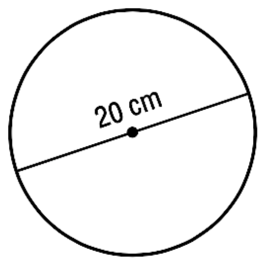

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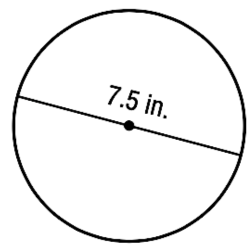
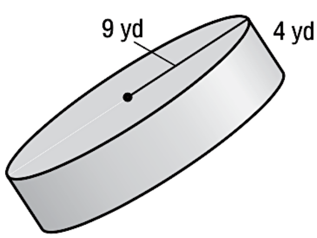


*Find the **SURFACE AREA** of each cylinder; use 3.14 for π, show all your work:*

7.



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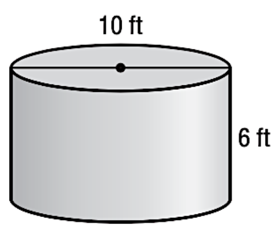

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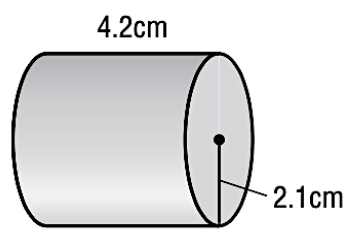
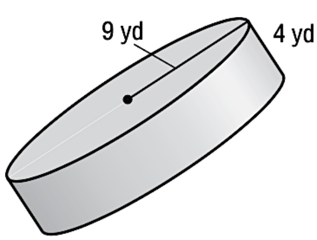
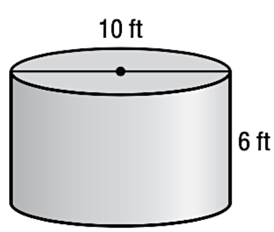


*Find the **VOLUME** of each cylinder; use 3.14 for π, show all your work:*

10.



11.



12.